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A Production Control Inventory Model for Continuous Deteriorating Items with Shortages and Two Different Rates of Productions

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ABSTRACT

In the present paper a production control inventory model for continuous deteriorating items with shortages. Here it is assumed that the production rate is changed to another at a time when the inventory level reaches a prefixed level Q_1 and continued until the inventory level reaches the level $S(>Q_1)$. Here we assume the demand rate as constant. The production is started again at a time when the shortage level reaches a prefixed quantity Q_2. In this model the total cost per unit time as a function of Q_1, Q_2 and S is derived. The optimal rules for Q_1, Q_2 and S are computed and the results are verified by numerical example.

INTRODUCTION

As most of the physical goods deteriorate, so in the recent years, the controls of production inventories of deteriorating items with shortages have received much attention of many researchers. In real life some of one items are either damaged or decayed or vaporized or affected by some other factors and are not in perfect condition to fulfill the demand.

Food items, drugs, pharmaceuticals, radioactive substances are examples of items in which sufficient deterioration can take place during the normal storage period of the units and consequently this loss must be taken into account when analyzing the system. Research in this direction began with the work of Within (1957) who considered fashion goods deteriorating at the end of a prescribed storage period. Ghare and Schrader (1963) developed an inventory model with a constant rate of deterioration. An order level inventory model for items deteriorating at a constant rate was discussed by Shah and Jaiswal (1977). Aggarwal (1978) reconsidered this model by rectifying the error in the work of Shah and Jaiswal (1977) in calculating the average inventory holding cost. In all these models, the demand rate and the deterioration rate were constants, the replenishment rate was infinite and no shortage in inventory was allowed.

Many researchers started to develop inventory systems allowing time variability in one or more than one parameters. Dave and Patel (1981) discussed an inventory model for replenishment. This was followed by another model by Dave (1986) with variable instantaneous demand, discrete opportunities for replenishment and shortages. Bahari-Kashani (1989) discussed a heuristic model with time-proportional demand. An Economic Order Quantity (EOQ) model for deteriorating items with shortages and linear tend in demand was studied by Goswami and Chaudhuri (1991). On all these inventory systems, the deterioration rate is a constant.

There are so many classes of inventory in the field of inventory control. A class of inventory models has been developed with time-dependent deterioration rate. Cover and Philip (1973) used a two-parameter Weibull distribution to represent the distribution of the time to deterioration. This model was further developed by Philip (1974) by taking a three-parameter Weibull distribution for the time to deterioration. This model was further developed by Philip (1974) by taking a three-parameter Weibull distribution for the time to deterioration. Mishra (1975) analyzed an inventory model with a variable rate of deterioration, finite rate of replenishment and no shortage, but only a special case of the model was solved under very restrictive assumptions. Deb and Chaudhuri (1986) studied a model with a finite rate of production and a time-proportional deterioration rate, allowing backlogging. Goswami and Chaudhuri (1992) assumed that the demand rate, production rate and deterioration rate were all time dependent.

Detailed information regarding inventory modelling for deteriorating items was given in the review articles of Nahmias (1982) and Raafat (1991). An order-level inventory model for deteriorating, items without shortages has been developed by Jalan Chaudhuri (1999). Ouyang et al. (1999) considered the continuous inventory system with partial backorders. A production inventory model with two rates of production and backorders is analyzed by Perumal and Arivarigan (2002).

Chang et al (2003) developed an EOQ model for deteriorating items under supplier credit linked to the ordering quantity. The deterministic inventory models were derived without derivatives by Huang (2003). Shortages are allowed with defective items. An order level inventory model for deteriorating items was developed by Sharma and Singh (2003). Chang (2004) discussed an inventory model for deteriorating items with stock dependent demand. The sensitivity of the inventory model with partial backorders was analyzed by Chu and Chung (2004). Chung and Liao (2004) investigated a lot sizing decision under trade credit depending on the ordering quantity.

Giri and Yun (2005) considered an economic manufacturing quantity problem for an unreliable manufacturing system where the machine is subject to random failure and at most two failures can occur in a production cycle. Lin and Lin (2005) developed an EOQ model for deteriorating items with linear demand and variable backlogging. Economic production quantity models for deteriorating items with price and stock dependent demand has been presented by Teng and Chang (2005).

Mana and Chaudhuri (2006) analyzed an EOQ model with ramp type demand rate, time dependent deterioration rate, unit production cost and shortages. Optimal and near optimal policies for lost sales inventory models with at most one replenishment order outstanding were presented by Hill and Johansen (2006). Inventory models based on the minimum cost approach. Pal et al (2006) developed optimal lot size model for deteriorating items with demand rate dependent on displayed stock level and partial backordering. Shortages are allowed and partially backlogged with a variable rate, which depend on the duration of waiting time up-to the arrival of next lot.

Chang et al (2006) discussed retailer's optimal pricing and lot sizing policies for deteriorating items with partial backlogging. Pricing is a major strategy for a retailer to obtain its maximum profit. This paper established an economic order quantity model for a retailer to determine its optimal selling price, replenishments number and replenishment schedule with partial backlogging. Chang et al (2006) developed an EOQ model for perishable items under stock dependent selling rate and time dependent partial backlogging. This paper establishes an appropriate model in which building up inventory is profitable, and provides an algorithm to find the optimal solution to the problem. Advanced planning with mixture of backorders and lost sales was developed by Lodree (2007). Teng et al (2007) discussed a comparison between two pricing and lot sizing models with partial backlogging for deteriorating items.

Cheng M and Wang G (2009) discused the inventory model for deteriorating items with trapezoidal type demand rate. Roy, A, (2008) discussed the inventory model for deteriorating items with price dependent demand and time varying holding cost. After this Skouri, K., Konstantaras, I., Papachristos, S., & Ganas, I. (2009) worked on Inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate. He, Y., Wang, S.-Y., & Lai, K.K. (2010) An optimal production-inventory model for deteriorating items with multiple-market demand Dye, C.-Y., & Ouyang, L.-Y. (2011). A particle swarm optimization for solving joint pricing and lot-sizing problem with fluctuating demand and trade credit financing. LeeaY.-P., & Dye, C.-Y (2012). An inventory model for deteriorating items under stock-dependent demand and controllable deterioration rate. Maihami, R., & Kamalabadi, I.N. (2012). Joint pricing and inventory control for non- instantaneous deteriorating items with partial backlogging and time and price dependent demand... Hill, R.M. and Johansen, S.G. (2013), Optimal and near optimal policies for lost sales inventory models with at most one replenishment order outstanding. European Journal of Operational Research, 169(1), 111-132.

In the present paper we have developed a production control inventory model for continuous deteriorating items with shortages in which two different rates of production are available and it is possible that production started at one rate and after some time it may be switched over to another rate. Such a situation is desirable in the sense that by starting at a low rate of production, a large quantum stock of manufactured item at the initial stage is avoided, leading to reduction in the holding cost.

NOTATIONS AND ASSUMPTIONS

The following notations and assumptions are used for developing the model.

(i) a is the constant demand rate.

(ii) $p_1(>a)$ And $p_2(>p_1)$ are the constant production rates started at time 0 and at time $t_1(>0)$.

(iii) C_1 is the holding cost per unit per unit time.

(iv) C_2 is the shortage cost per unit per unit time.

(v) C_3 is the cost of a deteriorated unit.

(C_1, C_2 and C_3 are known constants)

(vi) C is the average cost of the system.

(vii) $Q(t)$ is the inventory level at time $t(\geq 0)$.

(viii) Replenishment is instantaneous and lead time is zero.

(ix) T is the fixed duration of a production cycle.

(x) Shortages are allowed and backlogged.

(xi) The distribution of the time to deterioration of an item follows the exponential distribution $g(t)$ where

$$g(t)=\begin{cases}\theta e^{-\theta t}, & \text{for } t>0,\\ 0 , & \text{otherwise.}\end{cases}$$

θ is called the deterioration rate; a constant fraction $\theta(0<\theta<<1)$ of the on-hand inventory deteriorates per unit time. It is assumed that no repair or replacement of the deteriorated items takes place during a given cycle.

In this paper we consider a single commodity deterministic continuous production inventory model with a constant demand rate a. The production of the item is started at time 0, at a rate $p_1(>a)$, once the inventory level reaches Q_1, the rate of production is switched over to $p_2(>p_1)$ and the production is stopped when the inventory level reaches $S(>Q_1)$ and the inventory is depleted at a constant rate a. It is decided to backlog demands up to Q_2 which occur during stock out time. Thus the inventory level reaches $-Q_2$ (backorder level is Q_2), the production is started at a (faster) rate p_2 so as to clear the backlog and when the inventory level reaches 0 (i.e., the backlog is cleared), the next production cycle starts at the (lower) rate p_1.

We denote the duration of production at the rate p_1 by $[0,t_1]$, the duration of production at the rate p_2, by $[t_1,t_2]$, the duration when there is no production but only consumption by demand at a rate a, by $[t_2,t_3]$, the duration of shortage by $[t_3,t_4]$, and the duration of time to backlog at the rate p_2. by $[t_4,T]$. The cycle then repeats itself after time T.

THE MATHEMATICAL MODEL AND ITS ANALYSIS

Let $Q(t)$ be the on hand inventory at time $t(0 \le t \le T)$. then the differential equations describing the instantaneous states of $Q(t)$ in the interval $[0,T]$ are given by the following:

$$\frac{dQ(t)}{\partial t} + \theta Q(t) = p_1 - a, \qquad 0 \le t \le t_1 \qquad (1)$$

$$\frac{dQ(t)}{\partial t} + \theta Q(t) = p_2 - a, \qquad t_1 \le t \le t_2 \qquad (2)$$

$$\frac{dQ(t)}{\partial t} + \theta Q(t) = -a, \qquad t_2 \le t \le t_3 \qquad (3)$$

$$\frac{dQ(t)}{\partial t} = -a, \qquad t_3 \le t \le t_4 \qquad (4)$$

$$\frac{dQ(t)}{\partial t} = p_2 - a, \qquad t_4 \le t \le T \qquad (5)$$

The boundary conditions are:

$$Q(0)=0,\ Q(t_1)=Q_1,\ Q(t_2)=S,\ Q(t_3)=0,\ Q(t_4)=-Q_2,\ Q(T)=0 \qquad (6)$$

The solutions of equation (1)-(5) are given by

$$Q(t) = \frac{1}{\theta}(p_1 - 1)(1 - e^{-\theta t}), \qquad 0 \le t \le t_1 \qquad (7)$$

$$= \frac{1}{\theta}(p_2 - a) + e^{-\theta(t-t_1)}[Q1 - \frac{1}{\theta}(p_2 - a)], \qquad t_1 \le t \le t_2 \qquad (8)$$

$$= -\frac{1}{\theta}a + (S + \frac{1}{\theta}a)e^{-\theta(t-t_2)}, \qquad t_2 \le t \le t_3 \qquad (9)$$

$$= -a(t - t_3), \qquad t_3 \le t \le t_4 \qquad (10)$$

$$= -Q_2 + (p_2 - a)(t - t_4), \qquad t_4 \le t \le T \qquad (11)$$

From (6) and (7) we have

$$\frac{1}{\theta}(p_1 - a)(1 - e^{-\theta t_1}) = Q_1$$

$$\therefore\ t_1 = \frac{1}{\theta}\log[1 + \frac{\theta Q_1}{(p_1 - a)} + \frac{\theta^2 Q_1^2}{(p_1 - a)^2}] \text{ (Neglecting higher powers of } \theta, 0 < \theta << 1), \qquad (12)$$

Again from (6) and (8) we have

$$\frac{1}{\theta}(p_2 - a) + e^{\theta(t_1 - t_2)}[Q_1 - \frac{1}{\theta}(p_2 - a)] = S$$

$$\therefore \qquad e^{\theta(t_2 - t_1)} = [1 - \frac{S\theta}{(p_2 - a)}]^{-1}[1 - \frac{Q_1\theta}{(p_2 - a)}] \qquad (13a)$$

$$= 1 + \frac{(S - Q_1)\theta}{(p_2 - a)} + \frac{\theta^2 (S - Q_1)S}{(p_2 - a)^2} \text{ (Neglecting higher powers of } \theta) \qquad (13b)$$

$$\therefore \qquad t_2 = t_1 + \frac{1}{\theta}\log[1 + \frac{(S - Q_1)\theta}{(p_2 - a)} + \frac{\theta^2 (S - Q_1)S}{(p_2 - a)^2}] \qquad (14)$$

$$= \frac{1}{\theta}\log[1 + \frac{\theta Q_1}{(p_1 - a)} + \frac{\theta^2 Q_1^2}{(p_1 - a)^2}][1 + \frac{(S - Q_1)\theta}{(p_2 - a)} + \frac{\theta^2 (S - Q_1)S}{(p_2 - a)^2}] \text{ [by (12)]} \qquad (15)$$

From (9) we have

$$\frac{1}{\theta}a = (S+\frac{1}{\theta}a)e^{\theta(t_2-t_3)}, \qquad \text{Since } Q(t_3)=0$$

$$\Rightarrow \qquad t_3 = t_2 + \frac{1}{\theta}\log(1+\theta\frac{S}{a}) \tag{16}$$

$$\therefore \qquad t_3 = \frac{1}{\theta}\log(1+\frac{S\theta}{a})(1+\frac{\theta Q_1}{(p_1-a)}+\frac{\theta^2 Q_1^2}{(p_1-a)^2})[1+\frac{(S-Q_1)\theta}{(p_2-a)}+\theta^2\frac{(S-Q_1)S}{(p_2-a)^2},]$$

[by (15)] (17)

Again from (10) we have

$$a(t_3-t_4) = -Q_2, \text{ since } Q(t_4)=-Q_2 \tag{18}$$

$$\therefore \qquad t_4 = \frac{Q_2}{a}+\frac{1}{\theta}\log(1+\frac{S\theta}{a})(1+\frac{\theta Q_1}{(p_1-a)}+\frac{\theta^2 Q_1^2}{(p_1-a)^2})[1+\frac{(S-Q_1)\theta}{(p_2-a)}+\frac{\theta^2(S-Q_1)S}{(p_2-a)^2}]$$

[by (17)] (19)

From (11) and $Q(T)=0,$ we have

$$Q_2 = (p_2-a)(T-t_4) \tag{20}$$

Therefore total number of deteriorated items in $[0,T]$

$$=[(p_1-a)t_1+(p_2-a)(t_2-t_1)-S]+[S-a(t_3-t_2)]$$

$$=\frac{1}{\theta}(p_1-a)\log[1+\frac{\theta Q_1}{(p_1-a)}+\frac{\theta^2 Q_1^2}{(p_1-a)^2}]+\frac{1}{\theta}(p_2-a)\log[1+\frac{(S-Q_1)\theta}{(p_2-a)}+\frac{\theta^2(S-Q_1)S}{(p_2-a)^2}]$$

$$-a\frac{1}{\theta}\log(1+\frac{S\theta}{a})$$

$$=\frac{1}{\theta}(p_1-a)[\frac{\theta Q_1}{(p_1-a)}+\frac{\theta^2 Q_1^2}{(p_1-a)^2}-\frac{\theta^2 Q_1^2}{2(p_1-a)^2}]+\frac{1}{\theta}(p_2-a)[\frac{(S-Q_1)\theta}{(p_2-a)}+\frac{\theta^2(S-Q_1)S}{(p_2-a)^2}]$$

$$\frac{(S-Q_1)^2\theta^2}{2(p_2-a)^2}-\frac{1}{\theta}a[\frac{S\theta}{a}-\frac{S^2\theta^2}{2a^2}] \qquad \text{(Neglecting higher powers of } \theta\text{)}$$

$$=\frac{\theta}{2}[\frac{1}{(p_1-a)}Q_1^2+\frac{1}{(p_2-a)}(S^2-Q_1^2)+\frac{1}{a}S^2] \tag{21}$$

The deterioration cost over the period $[0,T]$

$$=\frac{1}{2}C_3\theta[\frac{Q_1^2}{(p_1-a)}+\frac{1}{(p_2-a)}(S^2-Q_1^2)+\frac{1}{a}S^2] \tag{22}$$

Shortage cost over the period $[0,T]$

$$=-C_2[-\int_{t_3}^{t_4}a(t-t_3)dt+\int_{t_4}^{T}[-Q_2+(p_2-a)(t-t_4)]dt] \qquad \text{[By (10) \& (11)]}$$

$$=\frac{C_2 p_2 Q_2^2}{2a(p_2-a)} \qquad \text{[By using (18) \& (20)]} \tag{23}$$

`The inventory carrying cost over the period $[0,T]$

$$=C_1[\int_0^{t_1}Q(t)dt+]\int_{t_1}^{t_2}Q(t)dt+\int_{t_2}^{t_3}Q(t)dt] \tag{24}$$

Now, $\int_0^{t_1} Q(t)dt = \frac{1}{\theta}(p_1 - a)\int_0^{t_1}(1-e^{-\theta t})dt$, [by (7)]

$$= \frac{1}{\theta}(p_1 - a)(t_1 + \frac{1}{\theta}e^{-\theta t_1} - \frac{1}{\theta})$$

$$= \frac{1}{2\theta}(p_1 - a)\theta(t_1^2 - \frac{1}{3}\theta t_1^3) \qquad \text{(Neglecting higher powers of } \theta\text{)}$$

$$= \frac{1}{2}(p_1 - a)[\{\frac{1}{\theta}\log[1+\frac{\theta Q_1}{(p_1-a)}+\frac{\theta^2 Q_1^2}{(p_1-a)^2}]\}^2 - \frac{1}{3}\theta\{\frac{1}{\theta}\log[1+\frac{\theta Q_1}{(p_1-a)}+\frac{\theta^2 Q_1^2}{(p_1-a)^2}]\}^3]$$

[by (12)]

$$= \frac{1}{2}\frac{Q_1^2}{(p_1-a)} + \frac{1}{3}\frac{\theta Q_1^3}{(p_1-a)^2} \qquad \text{(Neglecting higher powers of } \theta\text{)} \qquad (25)$$

$$\int_{t_1}^{t_2} Q(t)dt = \int_{t_1}^{t_2}[\frac{1}{\theta}(p_2 - a) + \{Q_1 - \frac{1}{\theta}(p_2 - a)\}e^{-\theta(t-t_1)}]dt \qquad \text{[by (8)]}$$

$$= \frac{1}{\theta}(p_2 - a)(t_2 - t_1) - \frac{1}{\theta}\{Q_1 - \frac{1}{\theta}(p_2 - a)\}\{e^{-\theta(t_2-t_1)} - 1\}$$

$$= \frac{1}{\theta^2}(p_2 - a)\log[1+\frac{(S-Q_1)\theta}{(p_2-a)}+\frac{\theta^2(S-Q_1)S}{(p_2-a)^2}+\frac{\theta^3 S^3(S-Q_1)}{(p_2-a)^3}]$$

$$-\frac{1}{\theta}\{Q_1 - \frac{1}{\theta}(p_2 - a)\{\frac{S\theta-(p_2-a)}{Q_1\theta-(p_2-a)}-1\} \qquad \text{[by (13)]}$$

$$= \frac{1}{\theta^2}(p_2 - a)[\frac{(S-Q_1)}{(p_2-a)}\theta+\frac{\theta^2(S-Q_1)S}{(p_2-a)^2}+\frac{\theta^3 S^2(S-Q_1)}{(p_2-a)^3}-\frac{1}{2}\{\frac{(S-Q_1)^2\theta^2}{(p_2-a)^2}+\frac{2(S-Q_1)^2 S\theta^3}{(p_2-a)^3}\}]$$

$$+\frac{1}{3}\{\frac{(S-Q_1)^3\theta^3}{(p_2-a)^3}\}] - \frac{1}{\theta}\{Q_1 - \frac{1}{\theta}(p_2-a)\}[\frac{\theta(S-Q_1)}{\theta Q_1-(p-a)}] \quad \text{(neglecting higher powers of } \theta\text{)}$$

$$= \frac{(S^2-Q_1^2)}{2(p_2-a)} + \frac{\theta(S^3-Q_1^3)}{3(p_2-a)^2} \qquad (26)$$

$$\int_{t_2}^{t_3} Q(t)dt = \int_{t_2}^{t_3}[-\frac{1}{\theta}a + (S+\frac{1}{\theta}a)e^{-\theta(t-t_2)}]dt \qquad \text{[by (9)]}$$

$$= -\frac{1}{\theta}a(t_3 - t_2) - \frac{1}{\theta}(S+\frac{1}{\theta}a)[e^{-\theta(t_3-t_2)} - 1]$$

$$= \frac{1}{\theta^2}a\log(1+\frac{S\theta}{a})^{-1} - \frac{1}{\theta}(-S) \qquad \text{[by (16)]}$$

$$= \frac{a}{\theta^2}[\frac{-S\theta}{a}+\frac{S^2\theta^2}{a^2}-\frac{S^3\theta^3}{a^3}-\frac{1}{2}(\frac{S^2\theta^2}{a^2}-\frac{2S^3\theta^3}{a^3})-\frac{1}{3}(\frac{S^3\theta^3}{a^3})]+\frac{S}{\theta}$$

(Neglecting higher powers of θ)

$$= \frac{S^2}{2a} - \frac{S^3\theta}{3a^2} \qquad (27)$$

Therefore the inventory carrying cost over the cycle $[0, T]$

$$=C_1[\frac{1}{2}\frac{Q_1^2}{2(p_1-a)}+\frac{1}{3}\frac{\theta Q_1^3}{(p_1-a)^2}+\frac{(S^2-Q_1^2)}{2(p_2-a)}+\frac{\theta(S^3-Q_1^3)}{3(p_2-a)^2}+\frac{S^2}{2a}-\frac{S^3\theta}{3a^2}] \quad (28)$$

From (20) we have

$$Q_2=(p_2-a)(T-t_4)=(p_2-a)[(T-t_3)-(t_4-t_3)]$$

$$=(p_2-a)[T-\frac{1}{\theta}\log(1+\frac{S\theta}{a})(1+\frac{\theta Q_1}{(p_1-a)}+\frac{\theta^2Q_1^2}{(p_1-a)^2})(1+\frac{(S-Q_1)\theta}{(p_2-a)})+\frac{\theta^2(S-Q_1)}{(p_2-a)^2}-\frac{Q_2}{a}]$$

[by using (17)&(18)]

$$=(p_2-a)[T-\frac{1}{\theta}(\frac{S\theta}{a}-\frac{S^2\theta^2}{2a^2}+\frac{\theta Q_1}{(p_1-a)}+\frac{\theta^2Q_1^2}{(p_1-a)^2})-\frac{\theta^2Q_1^2}{2(p_1-a)^2}+\frac{(S-Q_1)\theta}{(p_2-a)})+\frac{\theta^2(S-Q_1)}{(p_2-a)^2}$$

$$-\frac{(S-Q_1)^2\theta^2}{2(p_2-a)^2}-\frac{Q_2}{a}] \quad \text{(neglecting higher powers of } \theta\text{)}$$

$$\Rightarrow \quad Q_2=\frac{a(p_2-a)}{p_2}[T-\frac{1}{(p_2-a)}\{\frac{p_2S}{a}+\frac{Q_1(p_2-p_1)}{(p_1-a)}\}$$

$$-\frac{\theta}{2(p_2-a)^2}\{\frac{(2a-p_2)p_2S^2}{a^2}+\frac{Q_1^2(p_2-p_1)(p_1+p_2-2a)}{(p_1-a)^2}\}] \quad (29)$$

The average cost of the system [using (22), (23) & (28)]

$$=C(Q_1,S)=\frac{1}{2(p_2-a)T}[C_1\{\frac{(p_2-p_1)}{(p_1-a)}Q_1^2+\frac{p_2S^2}{a}\}+\frac{C_2p_2Q_2^2}{a}]$$

$$+\frac{\theta}{(p_2-a)T}[\frac{C_1}{3(p_2-a)}\{\frac{(p_2-p_1)(p_1+p_2-2a)Q_1^3}{(p_1-a)^2}+\frac{(2a-p_2)p_2S^3}{a^2}\}$$

$$+\frac{C_3}{2}\{\frac{(p_2-p_1)}{(p_1-a)}Q_1^2+\frac{p_2S^2}{a}\}] \quad (30)$$

Where Q_2 is given by (29)

The necessary conditions for $C(Q_1,S)$ to be minimum are

$$\frac{\partial C}{\partial Q_1}=\frac{(p_2-p_1)}{(p_1-a)(p_2-a)T}[(C_1Q_1-C_2Q_2)+\theta Q_1\{\frac{(p_1+p_2-2a)}{(p_1-a)(p_2-a)}\}(C_1Q_1-C_2Q_2)+C_3\}]=0 \quad (31)$$

$$\frac{\partial C}{\partial S}=\frac{p_2}{a(p_2-a)T}[\{1+\frac{\theta S(2a-p_2)}{a(p_2-a)}\}(C_1S-C_2Q_2)+C_3S\theta]=0 \quad (32)$$

Solving these and using (29) we get the optimal value Q_1^*,Q_2^* and S^* of Q_1,Q_2 and S respectively. Which minimize $C(Q_1,S)$ provided they satisfy the sufficient conditions

$$\frac{\partial^2 C}{\partial Q_1^2}=\frac{(p_2-p_1)}{(p_1-a)(p_2-a)T}$$

$$\left[\frac{C_2 a(p_2-p_1)}{p_2(p_1-a)}\left\{1+\theta\frac{Q_1(p_1+p_2-2a)}{(p_1-a)(p_2-a)}\right\}^2+\theta\frac{(p_1+p_2-2a)(2C_1Q_1-C_2Q_2)}{(p_1-a)(p_2-a)}+C_1+C_3\theta\right]$$

$$>0 \tag{33}$$

$$\left(\frac{\partial^2 C}{\partial Q_1^2}\right)\left(\frac{\partial^2 C}{\partial S^2}\right)-\left(\frac{\partial^2 C}{\partial Q_1 \partial S}\right)=\frac{(p_2-p_1)C_1}{a(p_1-a)^2(p_2-a)^2T^2}[C_1p_2(p_1-a)+C_2p_1(p_2-a)]+A\theta>0 \tag{34a}$$

where $A=$

$$\frac{(p_2-p_1)p_2}{a(p_1-a)(p_2-a)^2T^2}\left[\begin{array}{l}(C_1+C_2)\left\{\begin{array}{l}\dfrac{(p_1+p_2-2a)}{p_1(p_1-a)^2(p_2-a)}(2C_2aQ_1(p_2-p_1)+p_1(p_1-a)(2C_1Q_1-C_2Q_2))\\+C_3\end{array}\right\}\\+\left\{\dfrac{C_2a(p_2-p_1)}{p_2(p_1-a)}+C_1\right\}\left\{\dfrac{(2a-p_2)S(2C_1+C_2)}{a(p_2-a)}+C_3+\dfrac{C_3Q_2(p_2-2a)}{a(p_2-a)}\right\}\end{array}\right] \tag{34b}$$

If the solutions obtained from equations (31) and (32) do not satisfy the sufficient conditions (33) and (34), we may conclude that no feasible solution will be optimal for the set of parameter values taken to solve equations (31) and (32). Such a situation will imply that the parameter values are inconsistent and there is some error in their estimation.

NUMERICAL EXAMPLE

Equations (31) and (32) are now solved numerically (for different value of θ) with the help of a computer using the following values of the parameters:

$C_1=1, C_2=2, C_3=25, p_1=4, p_2=5, a=3, T=20.$

The optimal solutions are found to be (using (29))

θ	Q_1^*	S^*	Q_2^*	C^*
0.006	6.8082	17.7537	1.8786	8.4663
0.007	6.5244	18.1299	1.6296	8.9029
0.008	6.2840	18.4472	1.4134	9.3241
0.009	6.0772	18.7188	1.2225	9.7320
0.01	5.8972	18.9543	1.0515	10.1286

It is checked that these results satisfy the sufficient conditions (33) and (34) for minimizing C.

From this table it is clear that as θ increase, Q_1^* and Q_2^* decrease while S^* and C^* increase. It is also seen that S^* is slightly sensitive, Q_1^* and C^* are moderately sensitive while Q_2^* is highly sensitive to changes in the value of θ.

CONCLUSION

In this paper we have dealt with a continuous production inventory model for deteriorating items with shortages in which two different rates of production are available and it is possible that production started at one rate and after some time it may be switched over to another rate. Such a situation is desirable in the sense that by starting at a low rate of production, a large quantum stock of manufactured item at the initial stage is avoided, leading to reduction in the holding cost. The variation in production rate provides a way resulting consumer satisfaction and earning potential profit. For this model we have derived the average system cost and the optimal decision rules for Q_1, Q_2 and S when the deterioration rate θ is very small. Results are illustrated by numerical example.

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